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OPTIMIZATION THEORY AND APPLICATIONS

FINAL REPORT

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TABLE OF CONTENTS

Large Scale Linear Programming

A book is near completion which presents a comprehensive approach to large scale linear programming. A series of papers on the subject give the background for the book:

1. "Matrix Iterative Techniques In Large Scale Linear Programming," by Gary J. Koehler, Andrew B. Whinston, and Gordon P. Wright, Management Science, forthcoming.

In this paper, a brief summary is given for solving certain types of large scale linear programs by iterative methods. The advantages of such methods over traditional methods are expounded upon. Several classes of problems wherein iterative techniques appear useful as a solution procedure are discussed.

The primary method currently used for solving large scale linear programs is the Revised Simplex Method where the basis inverse is stored in one of many compact forms. Since the total number of iterations are fixed, savings in time and cost can be realized by performing each iteration efficiently. The major bottleneck in solving large scale linear programs has been in peripheral activity. Kalan observed that the reading of structural elements and storage and retrieval of the basis inverse are the major sources of time consumption in peripheral activity. With this in mind, Kalan pointed out that "the number of distinct values is small compared with the number of non-zero coefficients." This property, termed supersparsity, can be exploited to store the entire linear program in-core and to imbed much of the basis inverse in the structural elements of the problem.

Having eliminated the heavy demand on peripheral activity, there still remains the problem of storing a basis inverse efficiently, accurately, and effectively. This means that sparsity should be maintained, that arithmetic rounding errors should be negligible or easily handled, and that the

reinversion of the series of vectors representing the basis inverse be carried-out in a minimal amount of time.

In addressing the above problem, it was noted that large scale systems of equations are routinely solved by matrix iterative techniques and that such techniques inherently maintain sparsity, that arithmetic rounding errors pose no particular problem since the solution vector is only approximated to within a specified tolerance, and finally that the knowledge of a good solution can be used to accelerate convergence. It appears that iterative or indirect methods for solving systems of equations may offer distinct advantages to the solution of large scale linear programs if such methods could be exploited.

2. "Fixed Cost Minimization Over a Leontief Substitution System," by Gary J. Koehler, Andrew B. Whinston, and Gordon P. Wright, R.A.I.R.O., May 1974, V-2.

It has been pointed out in the literature that Bender's decomposition method appears useful as a solution strategy. Using Bender's method and the properties of a Leontief substitution system, a solution procedure is specified for solving fixed cost minimization problems over a Leontief substitution system. Preliminary results indicate that for some large problems, this procedure is a viable alternative to procedures specified elsewhere.

A Leontief substitution system is characterized by a constraint matrix having exactly one positive element per column and non-trivial rows. Substitute activities are identified by columns whose positive elements fall in the same row. A salient feature of Leontief substitution systems is that one and only one activity is optimal for each set of substitute activities.

Consider the problem

Min $f(x)$

subject to $Ax = B$

$x \geq 0$

$b \geq 0$

where A is m by n and Leontief, x is n by 1, b is m by 1, and $f(\cdot)$ is a quasi-concave function of the form:

$$f(x) = \sum_{i=1}^{n} f_i(x_i)$$

where

$$f_i(x_i) = \begin{cases} c_i x_i + d_i & x_i > 0 \quad d_i \geq 0 \\ 0 & x_i = 0 \end{cases}$$

The above problem is termed a fixed cost minimization over a Leontief substitution system.

3. "An Iterative Procedure for Nondiscounted Discrete-Time Markov Decisions," by Gary J. Koehler, Andrew B. Whinston, and Gordon P. Wright, Naval Research Logistics Quarterly, December 1974, Vol. 21, No. 4.

This paper utilizes the linear programming procedure given by Manne for finding gain optimal policies and the procedure given by Denardo for finding bias optimal policies which, by repeated application, gives policies optimal for all sufficiently small interest rates. The problem to be addressed is the efficient solution of the corresponding linear programs.

Consider the following linear program and its dual:

Primal: Max $c'x$	Dual: Min $b'\pi$
subject to:	subject to:
$Ax = b$	$A'\pi \geq c$
$x \geq 0$	$b \geq 0$
$b \geq 0$	

where A' is the transpose of A, A is m by n, b and π are m by 1, and x and c

are m by 1. Let J be a set of column identifications of A and A_J a submatrix of A consisting of the columns of A listed in J.

Consider a primal basis J. Let

$$A_J' = R_J - Q_J,$$

such that R_J is nonsingular and

(1) $x \geq y$ implies that $R_J^{-1}Q_J x \geq R_J^{-1}Q_J y$

(2) $p(R_J^{-1}Q_J) < 1$,

where $p(\cdot)$ is the spectral radius. If there exists an R and Q for each primal reasible basis J then iterative methods may be used to solve the dual linear program.

Mathematical Programming

1. "A Perturbation Theory Approach to Non-Linear Programming," by James Thurber and Andrew Whinston

The majority of problems which occur in mathematical physics are stated in the form of differential equations with some prescribed boundary and/or initial conditions. Generally, it is impossible to obtain closed form analytical solutions to such problems. A long standing method for obtaining approximate solutions is perturbation theory. One or more parameters enter the problem and for some fixed values of the parameters (usually taken to be zero) the problem is exactly solvable analytically. The equations and their solutions are expanded in terms of these parameters and one thereby obtains approximate solutions of the original problem.

We apply this method to non-linear mathematical programming, in order to obtain a sequence of approximate solutions which converges to solutions of the original problem. As a corollary to this approach, we are able to derive sufficient and/or necessary conditions for local optimum solutions

in a very general setting. In particular, when the normal first order conditions fail due to the presence of cusps, we derive a new set of sufficient conditions.

The perturbation series approach leads to a new algorithmic development which is discussed in this paper and will be further amplified in a sequel.

2. "Quasi-Concave Minimization Subject to Linear Constraints," by Antal Majthay and Andrew Whinston, Discrete Mathematics 9 (1974) 35-59.

A finite algorithm is presented for solving the quasi-concave minimization problem subject to linear constraints. The concept of an extreme point is generalized to that of an extreme facet of a polyhedron. Then a search routine is developed for the detection of an extreme facet of the feasible region relative to the polyhedron defined by the current set of cuts. After identifying an extreme facet, we cut it off by a cut developed for this purpose. We call this cut the facet cut. The method is both compatible with other cutting procedures and is finite.

Economic Theory

1. "Resource Allocation In a Non-Convex Economy," by James C. Moore, Andrew B. Whinston, and Joseph S. Wu, The Review of Economic Studies, Vol. 39 (3), July 1972.

This paper studies some of the questions of resource allocation without convexity assumptions on the production sets. Thus, we admit as possible the occurrence of increasing returns and indivisibilities in characterizing the available production technology. Undoubtedly admitting such possibilities adds to the realism of the models to be studied.

The principal subject to be studied in this paper is the relationship between Pareto optimality and competitive equilibria. Assuming the proper convexity conditions, it is well-known that these two basic concepts in economic theory are related. Recently, in a succession of papers, the con-

nection between a Pareto optimal solution and an associated competitive equilibrium without convexity has been discussed. By construction, it was shown that, despite the absence of convexity a competitive equilibrium was a possibility.

However, no general conditions were stated to characterize the problem of existence of a competitive solution to a given resource allocation problem; a gap in the theory which was noted by Koopmans in a summary paper on this topic. The principal intent of this paper is to help fill this gap. Specifically, we develop what we hope is a useful characterization of necessary and sufficient conditions for a Pareto optimal allocation to be attainable as a competitive equilibrium (in the case where the aggregate production set is not necessarily convex).

2. "A New Theory of Pricing and Decision-Making for Public Investment," by Edna Loehman and Andrew Whinston, The Bell Journal of Economics and Management Science, Vol. 2, No. 2, Autumn 1971.

This paper presents a theory for price-setting in public utilities, of which public goods will be considered a special case. Both public goods and utilities are cases of joint supplies and costs, where a good is to be supplied to and paid for by several users, and any increase in the quantity of the good is equally available to all users. The pricing system proposed will be based on the idea of each user's paying the social incremental costs due to his demands. To a certain extent, Coase proposed a similar idea, but it was not worked out except in special cases. The concept has been difficult to apply due to problems in defining incremental cost when there are joint costs. A meaningful definition of social incremental cost will be given here. Where the classical assumptions hold, our theory reduces to marginal-cost pricing; however, in the case of decreasing costs, results different from marginal-cost pricing are obtained. Since users may have

quite different demands, incremental-cost charges will not be uniform. However, the incremental-cost system does possess certain equity properties. Furthermore, this system or charges will cover the complete costs of supplying a public service. Finally, the concept of incremental cost implies that, at the optimum investment, the marginal unit should be charged marginal cost, which is the condition necessary for welfare maximization.

3. "Stochastic Control in Urban Economics," by James Thurber and Andrew Whinston.

The paper presents a planning model in the context of an urban economic setting. In effect, we will study an economy which has both space and time dimensions in terms of the state and control variables. We consider a radially symmetric region in which individual consumers will, over time, purchase land space and consumption goods and travel to a central business district (C.B.C.) Two sets of controls will be considered; first price function on transportation and land cost will be used as ways of equilibrating the market for these services. Secondly, a control in the form of an allocation of land between living space and roads is introduced. This control function affects the common level of utility for society.

We proceed by first extending a model of Solow to the time dependent case. The resulting model has the essential ingredients outlined above. Using that model, we derive the structure of the control process in terms of the control function rent, transportation costs, and road building. The original Solow model and our time dependent generalizations assume that a typical consumer will make his choices, represented by time function of consumption living space and location, in a completely rational way. However, we introduce inertia terms to reflect the fact that individuals have a certain reluctance to change their behavior. In the second part of the

paper, we alter the assumption of rationality by assuming only that there is a tendency towards rationality. In other words, we argue that from the point of view of a planning agency, the assumption of total rationality is unreasonable and that account of this should be taken in the determination of the controls. On the other hand, we assume the central authority will select the controls to maximize total utility of society. Thus we have a dual approach; individuals possessing only a tendency to maximize utility subject to the controls of the planning group but the planning group using this information choosing the controls to maximize total expected utility.

Accounting Information Systems

1. "A Structuring of An Events-Accounting Information System," by Arthur Z. Lieberman and Andrew B. Whinston, forthcoming, Accounting Review.

Information systems have traditionally been designed so that each department or section of a company generates and maintains its own data files. Some drawbacks of this method are obvious: data is at times replicated through several departments, it may be difficult to accumulate common data accurately from the departments, and it would be difficult for other people in management to access quickly some particular data from any individual department. There are other inconveniences of allowing (or, in fact, or requiring) each department to maintain its own data files, including the need for individualized programs to process the data that may enter the system in varied formats.

The concept of the common data base brings together all of this data into one central set of files, a set which eliminates the duplication of data and allows for quick and easy access to any item. This data base, along with the routines for processing the data, comprises an information

system, and the success, or value, of the information system is a function of how easily and economically management can get the information it needs. The desired information might not be just raw data as originally stored in the data base; it may involve computations and conditional processing of the data.

This writing proposes a "total" information system based on the "events" approach to accounting (Sorter, 1969). The suggested system is more than just an accounting information system helping the accountant to collect and report his figures; the proposed system would provide all desired data and information (even that not normally provided by the fiscal accountant or not directly applicable to the accounting function) which might be helpful to all levels of management in the decision-making processes -- thus the term "total" information system.

Events accounting is an approach in which recording of the data corresponds to the occurrence of economic events affecting a firm. The concept of events accounting is still in its developmental stage; its meaning and structure is defined in the writings of only a few individuals. Here, in the process of structuring an "Events-Accounting Information System," support is given to the meaning of events accounting.

"System" refers to the interrelations among 1) the computer (hardware); 2) its programs (software); 3) the concepts upon which the design of the hardware and software are based; and 4) managers (the users to whom the computer is but a tool in their decision-making process). "Structure" refers to both the conceptual and the physical methods of representing and storing the data to be used in the system.

Pattern Recognition

1. "Application of Pattern Recognition to Some Problems In Economics," by Jean-Marie Blin, King-Sun Fu, and Andrew B. Whinston, Techniques of Optimization, Ed. A. V. Balakrishnan.

Economic theory, as it stands today, displays a rather appalling contrast between the level of sophistication attained by micro economic theory on the one hand and "public economics" on the other hand. As a matter of fact, the very term "public economics" is vague enough to warrant some further specification. Micro economic theory has given us a very general mathematical model of the production and consumption of private goods. These goods are such that their consumption (or the consumption of their services) by one individual excludes the consumption of the same goods by another individual. Generally speaking, they are divisible (or, at least, their services are) and they are privately appropriated -- at least in most western countries.

A moment's reflection will remind us of the existence of a very different kind of commodity, viz, "public goods." These are usually indivisible commodities, jointly consumed by a large number of individuals who may or may not pay an equal price for their services, depending upon the nature of the good and its particular financing. The provision of public goods is not an individual decision but rather a collective one. The elaborate models of production and consumption of private goods are no longer applicable or, to put it another way, the decision rules of utility maximization for the individual consumer and profit maximization for the individual producer are of no help since what we need is a collective decision rule. The study of such collective decision rules is actually two-fold: on the one hand, one may ask how such a rule happens to be - or ought to be - chosen. In other words, what rule can we use to decide upon a rule? This problem, which has the flavor of an infinite regress, is known in the literature as the "constitu-

tional choice problem." On the other hand, although the concepts to be outlined below have been quite useful in its solution, we shall concentrate here on a logically posterior problem, viz. the choice of a collective decision rule by some outside "advisor" on the basis of various rationality requirements. This is the classical problem of "individual preference aggregation" in Economics. The basic aim of this study is to show how one can use a single underlying structure, viz. that of a preference pattern, and its various mathematical representations to tackle the problem of social choice. The use of classical methods of pattern recognition leads to (1) a completely general concept of collective decision rule and (2) actual processes for collective decision-making. Our study will proceed as follows. First of all, in order to motivate our discussion, we shall briefly review the aggregation problem for consumer preferences and then translate its formulation in terms of pattern recognition which will give us a very clear understanding of the so-called "voting paradox." Then we will relate some classical methods of pattern recognition with the notion of a collective decision rule. And finally, we will present two possible aggregation algorithms for binary preference patterns.

2. "Discriminant Functions and Majority Voting," by J. M. Blin and Andrew B. Whinston, Management Science, forthcoming.

This paper is based on the notion of discriminatory power of a pattern classifier acting over a feature space. Discriminant functions are normally used to trace out regions of the feature space corresponding to specific pattern classes. Indeterminate cases occur whenever a pattern lies on a class boundary. The number of occurrences of such indeterminacies measures the relative discriminatory power of a family of discriminant functions. In this paper, these concepts are applied to majority voting decisions: it is first shown how regular majority voting leads to discriminant functions with

insufficient discriminatory power whenever intransitive social ordering occur. A new (improved) family of discriminant functions is proposed to resolve such cases.

3. "Social Choice and Formal Language Theory," by Mary Louise Piccoli and Andrew Whinston, Journal of Cybernetics, forthcoming.

The aggregation of individual preference orderings into group orderings or decisions has been extensively studied by economists and political scientists. The formal theory attempts to explain and justify group choice mechanisms such as voting rules and judicial processes. An aim of a democratic process, for example, is to develop procedures that allow individual preferences to be reflected in the final result. The formal development has employed mathematical techniques to either describe the process or to deny the possible existence of processes satisfying given criteria (axioms) of fairness. The attention has ranged from models that study fairly abstract binary choice problems to more realistic and complex phenomena such as vote-trading, strategic voting, and nonbinary choice described by choice functions.

Formal language and automata theory are specifically designed to model processes and restrictions imposed on those processes and so seem especially useful in generalizing present theories of social choice. In this paper, formal language and automata theory are used. The processes themselves, the interdependencies which characterize economics, are modeled. No assumptions of functional forms or mathematical dependence are necessary, in contrast to the mathematical models traditionally used in economics.

We begin by introducing the language and automata concepts necessary and show their ability to model a simple social choice process described by Farquharson and the common motion voting procedure. The general social choice process will be completely factored into its component subprocesses

modeled by formal language and automata theory. It will also be shown that the axiomatic approach to social choice is a special case of our system. Thus, applications of formal language and automata theory will be shown to generalize both the structural and axiomatic views of social choice, including the choice function approach.

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